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ANALYSIS OF THE LIGHT CURVES AND DETERMINATION OF THE ORBITAL ELEMENTS FOR THE ECLIPSING VARIABLES KR LAC , MQ CEP , GS LAC , LW LAC

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ABSTRACTS

ANALYSIS OF THE LIGHT CURVES AND DETERMINATION OF THE ORBITAL ELEMENTS FOR THE ECLIPSING VARIABLES KR LAC , MQ CEP , GS LAC and LW LAC .

This paper discusses the photographic light curves of the eclipsing binary stars KR Lac , MQ Cep , GS Lac and LW Lac, so as to determine their photometric orbital elements and their absolute elements. Starting from the observations available, the four orbits were determined first, by means of an iterative process fitting the curve against time.
All the calculated orbits are of the circular type and the determined elements are :

KR Lac (annular eclipse at principal minimum) k = .75 ± .01 xg = .51 xs = .57
rg = .423 ± .001 rs = .316 ± .006 |i| = 86°.1 ± 1°.6 Lg = .81 ± .02 Ls = .19 ± .02 Jg/Js = 2.36 ± .06

MQ Cep (partial eclipse, occultation at principal minimum) k = .67 ± .01 xg = .44 xs = .53
rg = .381 ± .002 rs = .257 ± .006 |i| = 78°.2 ± 0°.9 Lg = .312 ± .009 Ls = .688 ± .009 Jg/Js = .21 ± .02

GS Lac (annular eclipse at principal minimum) k = .95 ± .02 xg = .24 xs = .27
rg = .349 ± .004 rs = .331 ± .012 |i| = 90°.0 Lg = .86 ± .02 Ls = .14 ± .02 Jg/Js = 5.29 ± .09

LW Lac (total eclipse at principal minimum) k = .57 ± .02 xg = .60 (assumed) xs = .60 (assumed)
rg = .201 ± .007 rs = .114 ± .008 |i| = 90°.0 Lg = .143 ± .003 Ls = .857 ± .003 Jg/Js = .05 ± .05

The determination of the absolute elements has been made in two different ways :

1) in the case of KR Lac, a statistical procedure was used, taking in consideration empirical laws as the Period-Temperature, Mass-Radius and Mass-Temperature relations and giving, after combination with the photometric orbital elements, the following results :

KR Lac	Tg = 10 900 °K	Ts = 10 900 °K	mg = 3.4 M☉	ms = 2.1 M☉
	Rg = 2.4 R☉	Rs = 1.9 R☉	a = 5.67 R☉	Sp = B9 + B9

2) in the case of MQ Cep and GS Lac, on the other hand, it was supposed that the more massive component belonged to the main sequence, thus allowing the use of an iterative process taking in consideration the Mass-Radius, Mass-Temperature relations, Kepler's Third Law and the relation :

Js/Jg = (exp(-ws) - 1)/(exp(-wg) - 1) ws = 1.439/(λ x Ts) wg = 1.439/(λ x Tg)

This resulted in a convergence on the following set of parameters which has proved to be satisfactory:

MQ Cep	Tg = 11 900 °K	Ts = 22 900 °K	mg = 4.5 M☉	ms = 4.6 M☉
	Rg = 3.2 R☉	Rs = 2.1 R☉	a = 8.4 R☉	Sp = B3 + B8
GS Lac	Tg = 19 900 °K	Ts = 10 500 °K	mg = 11.7 M☉	ms = 10.6 M☉
	Rg = 5.8 R☉	Rs = 5.5 R☉	a = 16.7 R☉	Sp = B3 + B8

In the case of LW Lac, however, it has not been possible to obtain any absolute element on account of the unsatisfactory quality of the observations.
The paper concludes on the need for new and more thorough observations of these stars.

RESUME. Dans la présente étude on a analysé les courbes photographiques des binaires à éclipses KR Lac, MQ Cep, GS Lac et LW Lac, pour déterminer les éléments photométriques de l'orbite et les éléments absolus. Les 4 orbites - circulaires - ont tout d'abord été calculées par une méthode itérative de "curve fitting". Ensuite les éléments absolus ont été déterminés par 2 méthodes itératives différentes, l'une pour KR Lac, et l'autre pour MQ Cep et GS Lac. Pour LW Lac en revanche, il n'a pas été possible d'obtenir les éléments absolus, en raison de la qualité insuffisante des observations de départ.
Il serait souhaitable que ces étoiles puissent être réobservées de façon plus suivie.

Nota: les valeurs numériques obtenues pour les éléments figurent dans le résumé en anglais.

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RESUMEN. En el presente trabajo se analizan las curvas de luz fotográficas de las variables a eclipses KR Lac, MQ Cep, GS Lac y LW Lac, con el fin de determinar sus elementos orbitales fotométricos y sus elementos absolutos.
En base a las observaciones disponibles, han sido determinadas las 4 orbitas mediante un proceso iterativo de "curve fitting" en el dominio temporal.
Todas las órbitas calculadas son de tipo circular, y los elementos determinados son :

(eclisse anulare al minimo principale)

KR Lac
$k = .75 \pm .01$ $xg = .51$ $xs = .57$
$rg = .423 \pm .001$ $rs = .316 \pm .006$ $|i| = 86°.1 \pm 1°.6$
$Lg = .81 \pm .02$ $Ls = .19 \pm .02$ $Jg/Js = 2.36 \pm .06$

(eclisse parziale occultazione al minimo principale)

MQ Cep
$k = .67 \pm .01$ $xg = .44$ $xs = .53$
$rg = .381 \pm .002$ $rs = .257 \pm .006$ $|i| = 78°.2 \pm 0°.9$
$Lg = .312 \pm .009$ $Ls = .688 \pm .009$ $Jg/Js = .21 \pm .02$

La determinación de los elementos absolutos ha sido efectuada de dos formas :
1) en el caso de KR Lac ha sido utilizado un procedimiento de tipo estadístico explotando las leyes empíricas periodo-temperatura, masa-radio y masa-temperatura que, en combinación con los elementos orbitales fotométricos, han dado el siguiente resultado:

KR LAC $Tg = 10900$ °K $Ts = 10900$ °K $mg = 3.4$ MΘ $ms = 2.1$ MΘ
$Rg = 2.4$ RΘ $Rs = 1.9$ RΘ $a = 5.67$ RΘ Sp = B9+B9

2) en el caso de MQ Cep y GS Lac se ha supuesto que la componente más masiva pertenece a la secuencia principal, por lo cual, en base a un proceso iterativo, utilizando las leyes masa-radio, masa-temperatura, la 3ª Ley de Kepler y la relación :

$$Js/Jg = (\exp(-ws) - 1)/(\exp(-wg) - 1)$$

se ha logrado converger bajo la siguiente combinación de parámetros que se ha mostrado satisfactoria :

MQ Cep $Tg = 11900$ °K $Ts = 22900$ °K $mg = 4.5$ MΘ $ms = 4.6$ MΘ
$Rg = 3.2$ RΘ $Rs = 2.1$ RΘ $a = 8.4$ RΘ Sp = B3+B8

GS Lac $Tg = 19900$ °K $Ts = 10500$ °K $mg = 11.7$ MΘ $ms = 10.6$ MΘ
$Rg = 5.8$ RΘ $Rs = 5.5$ RΘ $a = 16.7$ RΘ Sp = B3+B8

En el caso de LW Lac, no ha sido posible obtener ningún elemento absoluto debido a la mala calidad de las observaciones disponibles.
Por último, se propone que esta estrella pueda ser observada nuevamente y de manera más completa.

RIASSUNTO. Nel presente lavoro vengono analizzate le curve di luce fotografiche delle variabili ad eclisse KR Lac, MQ Cep, GS Lac e LW Lac al fine di determinare gli elementi orbitali fotometrici egli elementi assoluti.
In base alle osservazioni disponibili sono state determinate le 4 orbite mediante un processo iterativo di "curve fitting" nel dominio temporale.
Tutte le orbite calcolate sono di tipo circolare e gli elementi determinati sono :

(eclisse anulare al minimo principale)

GS Lac
$k = .95 \pm .02$ $xg = .24$ $xs = .27$
$rg = .349 \pm .004$ $rs = .331 \pm .012$ $|i| = 90°.0$
$Lg = .86 \pm .02$ $Ls = .14 \pm .02$ $Jg/Js = 5.29 \pm .09$

(eclisse totale al minimo principale)

LW Lac
$k = .57 \pm .02$ $xg = .60$ (assunto) $xs = .60$ (assunto)
$rg = .201 \pm .007$ $rs = .114 \pm .008$ $|i| = 90°.0$
$Lg = .143 \pm .003$ $Ls = .857 \pm .003$ $Jg/Js = .05 \pm .05$

La determinazione degli elementi assoluti è stata effettuata in due modi :
1) nel caso di KR Lac è stata seguita una via di tipo statistico sfruttando le leggi empiriche Periodo - Temperatura, Massa - Raggio e Massa - Temperatura che in combinazione con gli elementi orbitali fotometrici hanno fornito i seguenti risultati :

2) nel caso invece di MQ Cep e GS Lac è stato assunto che la componente più massiva appartenga alla sequenza principale quindi in base ad un processo iterativo, usando le leggi Massa - Raggio, Massa - Temperatura, la 3ª legge di Keplero e la relazione :

$$ws = 1.439/(\lambda \times Ts) \quad wg = 1.439/(\lambda \times Tg)$$

è stata raggiunta la convergenza sulla seguente combinazione di parametri che si è dimostrata soddisfacente :

Nel caso di LW Lac non è stato possibile ottenere nessun elemento assoluto a causa della cattiva qualità delle osservazioni di partenza.
Si auspica alla fine che queste stelle possano essere osservate nuovamente e in maniera più completa.

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1. INTRODUCTION:

The stars under discussion were discovered during a photographic search for new variable stars in the constellations Cepheus and Lacerta, made in 1948 by W.J. MILLER and A.A. WACHMANN.

The number of observations available for each of the four stars is:

KR LAC: 308
MQ CEP: 354
GS LAC: 365
LW LAC: 368

These observations were published in 1971 (1) and are all photographic. The same paper also gives data about instruments and the photographic techniques used.

The general data concerning the four stars are as follows:

star	α (1900)	δ (1900)	type	m0	m1	m2
KR LAC	$22^h\ 11^m\ 48^s$	+50° 34' 05"	EB	15.42	16.29	15.81
MQ CEP	22 03 22	54 11 25	EB	15.38	16.60	15.72
GS LAC	22 22 24	52 45 41	EA	15.06	16.98	15.23
LW LAC	22 18 07	50 42 57	EA	14.05	16.20	14.08

The published ephemerides relative to the primary minimum are:

KR LAC 2434001.334 + $0^d.9376954$ xE
MQ CEP 2434152.609 + 0.9305391 xE
GS LAC 2433271.329 + 1.6771710 xE
LW LAC 2432768.160 + 14.3963 xE

The times are all heliocentric.

The spectral types for the components of the four systems are unknown. To date, it does not appear that any of these stars has been observed further, nor any orbit calculated for them. This present study follows that published on IW CEP (2) and is part of a programme under way at the C.A.B.. This programme plans to study eclipsing binaries whose magnitudes are too faint for photoelectric observations but which are otherwise documented by photographic observations with a precision sufficient to allow a calculation of the orbit.

2. OBSERVATIONS:

The stars under discussion were observed photographically between 1948 and 1955 at the Vatican Astronomical Observatory and at the Hamburg Observatory by W.J. MILLER and A.A. WACHMANN in the course of a programme of systematic search for new variables in the constellations CEP and LAC. According to the indications supplied by the authors, the instrument was a 40 cm Zeiss astrograph. The plates used were of various types, but it can be admitted with sufficient accuracy that λ_{eq} = 4250 Å. Moreover, the authors give a mean uncertitude of ± 0.03 mag for the points plotted in the published curves. The observations can therefore be considered as accurate. The four curves availa ble show an optimal coverage of the period of observation, a fact which increases the reliability of the results.

3. OBTAINING DATA:

The ephemerides reported in the literature and given in the table above are discretely reliable except in the case of LW Lac. For this latter star, the authors did not give the number of minima which they used to get the curve. Moreover, the authors did not give the error bars for the base and the period of the ephemerides they determined. Last of all, the method they used to obtain the instants of minima is not known and, in a like manner, nothing is said about the error bars for these instants. In any case, the measures they used to determine each instant are not very numerous and as a consequence, it can be supposed that the mean incertitude may be fairly important.

From the table given in the literature (1), it is possible to plot a mean light curve. A Texas DX 990 computer and a PRINTRONIX 300 printing unit were used to that purpose.

The light curves thus obtained were used, before correction, for the calculation of the orbits: they are shown in figs. 1,2,3,4. All the curves have been plotted according to the ephemerides listed above and the coordinates of the points are given in the tables attached to each curve. From an analysis of the 4 curves, the following results have been derived for the magnitude at maximum and at the primary and secondary minima. For the stars KR LAC, MQ CEP and GS LAC the value accepted for the error bars is that given on the average for each magnitude determined by the authors in the current literature.

	max	min1	min2
KR LAC	15.41 ±.03	16.29 ±.03	15.80 ±.03
MQ CEP	15.38 ±.03	16.60 ±.03	15.72 ±.03
GS LAC	15.06 ±.03	16.98 ±.03	15.23 ±.03
LW LAC	14.07 ±.02	16.20 ±.04	14.08 ±.03

The calculation of the mean minima gave the following values, obtained by a successive resolution of polygons (polygon method), for φ_1 and φ_2. (3)

	φ 1	φ 2	$\varphi 2 - \varphi 1$
KR LAC	.997±.006	.51 ±.02	.51 ±.02
MQ CEP	.002±.001	.499 ±.008	.497±.008
GS LAC	.003±.004	.507 ±.004	.504±.006
LW LAC	.996±.003	.544 ±.002	.548±.004

An examination of the values of $\varphi 1$ and $\varphi 2$, and of the difference in phase between the two minima leads to the following conclusion: the error bars given above only reflect the precision of the geometric method used to determine the minima and they must be considered as a sign of asymmetry between the two branches of the minima. It is therefore possible that the real error bars could well be higher than the given values.

It is then quite possible to assume a circular orbit for each of the four stars and consequently, such orbits will be calculated.

4. SOLUTIONS FOR THE LIGHT CURVES:

From the light curves plotted with the points listed in the tables, the photoelectric orbital elements have been determined, before correction of the light curve itself in the parts not affected by the eclipse. These elements were determined by means of a series of the type;

$$I(\theta) = A_0 + A_1 \cos\theta + A_2 \cos^2\theta \qquad (1)$$

neglecting the perturbation terms $B_j \sin^j\theta$ whenever the curve showed a good symmetry and in the parts not affected by eclipse. The values thus obtained for the four stars using the Binnendijk method are listed below:

	A_0	A_1	A_2
KR LAC	1.153 ± .007	−.016 ± .016	−.49 ± .09
MQ CEP	.998 ± .01	−.019 ± .001	−.196 ± .012
GS LAC	1.010 ± .02	−.010 ± .06	−.02 ± .25
LW LAC	1.0093± .0000	−.014 ± .008	−.028 ± .007

These constants enabled a corrected light curve to be plotted and this was used for the calculation of the orbits. The light curves of fig 1,2,3,4 show the following characters:

KR LAC: the primary eclipse is of annular type and the secondary of the total type, that is, at min I, the larger star, which is also the brighter, is eclipsed, whereas at minimum II, the contrary occurs. The light curve is typical of a system in contact and perturbations are present on practically the whole light curve.

MQ CEP: the light curve is well defined, with an excellent symmetry and it is practically devoid of perturbations. The EB type seems well confirmed by the curve. The primary eclipse appears to be partial, caused by the occultation of the more luminous, smaller star by part of the larger, darker component; the secondary minimum is caused by a transit of the smaller component across the larger one.

GS LAC: the primary minimum is caused by an annular eclipse (transit) of the larger, more luminous star by the smaller, less bright component. The secondary minimum is due to an occultation. The type suggested by the curve is EA and the perturbations are rather small; moreover, the symmetry is fairly good.

LW LAC: the primary minimum is rather deep and not well covered by observations, which entails some uncertainty in the solution of the light curve. On the other hand, the secondary minimum is better defined but extremely flattened. All in all, the curve is difficult to analyse. The primary minimum is caused by an occultation of the smaller yet brighter star and this minimum is of the total type. The secondary minimum corresponds to an annular eclipse. For the rest of the curve, there is a good symmetry without notable perturbations, which strongly suggests an EA type.

As a rule, only annular eclipses can lead to a reliable determination of the limb darkening coefficient; however, if the light curve is not too perturbed, it is possible to assume, by means of a relevant adjustement process, the values of coefficients xg and xs for the components, and this even in the case of a partial eclipse.

This process was applied to KR LAC, GS LAC, and MQ CEP for which the convergence was obtained fairly quickly, whereas in the case of LW LAC it was not possible to get values for these coefficients. In this latter case, it was necessary to set values a priori for them. At this stage, it must be noted that, as the spectral types were not known, it was not possible to make a preliminary choice of some reasonable value for xg and xs; it was therefore necessary to calculate a greater number of solutions, varying each time the value for the limb darkening coefficient.

In each case, however, it was assumed that the disk brightness of each component was given by the well-known law:

$$I_\lambda(g,\gamma) = I_\lambda(g_0)(1+\beta_\lambda/4 - \beta_\lambda/4.g/g_0)(1-x_\lambda + x_\lambda \cos\gamma) \qquad (2)$$

where x_λ is the limb darkening coefficient and β_λ the gravity darkening coefficient .

Two iterative methods were used to solve the four light curves, one applying to the partial eclipses and the other to the total eclipses, both in "time domain", in the version proposed by Gaspani at the 1980 GEOS Symposium (5) and at the Convention of Italian Variable Star Observers at Siena (1981)(6).

For MQ CEP, the orbital elements were obtained by using the points corresponding to the phase of the eclipse and by adjusting them so as to minimize the function defined by:

$$S(\alpha,k,x) = \sum_{j=1}^{j=n} \left\{ \sin^2\theta_j - G_1(k,x).D_j(\alpha,k,x)+G_2(k,x) \right\}^2 \qquad (3)$$

where G_1 and G_2 are constants linked to the orbital elements obtained for each value of k and x set by a least square method; j refers to the j^{th} point and

$$D_j(\alpha,k,x)=(1+k.\rho_j(\alpha,k,x))^2 \qquad (4)$$

In the case of KR LAC, GS LAC and LW LAC, the same process was used but for the fact that the function to minimize was:

$$S(\alpha,k,x)=\sum_{j=1}^{j=n} \left\{ \sin^2\theta_j - G_1(k,x).(D_j(\alpha,k,x)-D_0(k))-C_0(k,x) \right\}^2 \qquad (5)$$

where D_j is defined in the same manner as above (4) and $D_0(k)$ by:

$$D_0(k)= (1-k)^2$$

Thus obtaining a certain number of solutions for varied values of k and x, better values for these two parameters were then sought, that is to say values corresponding to a minimum of the function $S(\alpha,k,x)$. In general, the convergence was obtained only if the curve showed little perturbation at the minimum under analysis.

It was not possible to obtain any convergence in the case of LW LAC, hence no value of xg and xs could be obtained for this star. For the other stars, on the contrary, the convergence was good and the results satisfying.

Barring the case of LW LAC, the three absolute minima of the function $S(\alpha,k,x)$ turned out to be:

	k_O	x_O
KR LAC	.7459	.5056
MQ CEP	.6750	.4427
GS LAC	.9471	.2405

These are the pairs of values which fit the experimental data best. All calculations were performed with the help of a hand-held programmable calculator TEXAS TI-58C using a relevant programme. For all the calculations, it was assumed that the orbits were circular.

The results obtained are given in table I and figures 1,2,3,4 show the theoretical curves plotted from the experimental data; the agreement is good and the elements can therefore be considered as reliable.

By analyzing the orbital elements thus obtained, it is then possible to draw the following conclusions:

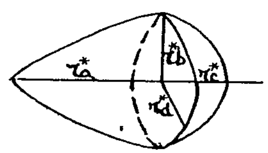
KR LAC: from an examination of the fractional radii and of the light curve, it can be inferred that the system under discussion is a contact system in which both components fill their own Roche lobe. In this case, it appears necessary to reconstruct the true shape of the two stars composing the system. This was done, assuming the following were valid:

1/ The two components have reached radiative balance.
2/ Both stars are considered as point-masses.
3/ The orbits are circular.
4/ The axial rotation is synchronous with a Keplerian rotation.
5/ The radiation pressure does not affect significantly the critical surface.

These conditions being set, the two Roche surfaces were determined: to this effect, special polynomial series developed by Gaspani (6) were used, which made the determination of the requested parameters speedier and easier.

These calculations gave the values for the fractional radii listed in the table below and defined in the figure attached to the table.

	main star	companion
r^*_a	.551 ± .001	.460 ± .009
r^*_b	.421 ± .001	.330 ± .006
r^*_c	.421 ± .001	.330 ± .006
r^*_d	.398 ± .001	.316 ± .006



If the hypothesis of a contact system is valid, the resulting mass ratio is then:

$$q = \frac{m_s}{m_g} = .610 \pm .001$$

MQ CEP: From the analysis of the fractional radii, it appears fairly evidently that the star is a semi-detached system (SD). The component with the greater radius fills its Roche lobe. Assuming the hypotheses previously made for KR LAC were valid, it was possible to calculate the shape of this component. Taking in account the mass ratio in the system (supposing the SD system type to be correct), it was then possible to determine the shape of the main star. The fractional radii thus derived are given in the table below:

	main star	companion
r^*_a	.263 ± .006	.503 ± .003
r^*_b	.252 ± .006	.375 ± .002
r^*_c	.263 ± .006	.375 ± .002
r^*_d	.256 ± .006	.357 ± .002

Considering the fact that the secondary component fills its Roche lobe, it is then possible to derive the mass ratio:

$$q = \frac{m_g}{m_s} = .939$$

GS LAC: A study of the light curve shows that this system is evidently of the detached type.

The fractional radii of the two components are:

	main star	companion
r^*_a	.367 ± .004	.348 ± .013
r^*_b	.333 ± .004	.316 ± .011
r^*_c	.367 ± .004	.348 ± .013
r^*_d	.347 ± .004	.329 ± .012

The calculated mass ratio, assuming a detached system, is;

$$q = \frac{m_s}{m_g} = .905$$

LW LAC: An analysis of the light curve shows that the system is of detached type. The fractional radii of the components are

	main star	companion
r^*_a	.202 ± .007	.114 ± .008
r^*_b	.200 ± .007	.114 ± .008
r^*_c	.202 ± .007	.114 ± .008
r^*_d	.201 ± .007	.114 ± .008

the derived mass ratio is then:

$$q = \frac{m_s}{m_g} = .480$$

5. DETERMINATION OF THE ABSOLUTE ELEMENTS:

The absolute elements for KR LAC, on the one hand, and MQ CEP and GS LAC, on the other hand, were derived in two different ways. As for LW LAC, however, it was not possible to find any reasonable solution to the problem.

KR LAC: As seen previously, such a system is of the "C" type: it is therefore possible to derive absolute parameters with the help of some statistics. Considering a sample of 20 systems (7), a correlation between the orbital period and the temperature of the brighter (and

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larger) component can be determined. This correlation is shown in fig. 5 and can be expressed tolerably well by:

$$\log T_1 = 4.06 + .66 \times \log P \qquad (7)$$

where P is in days and T_1 in °K.

This relation was used to determine the temperature of the main component. Moreover, the relation below is well verified for the 20 binary systems of the sample:

$$\log m_1 = -6.93 + 1.85 \times \log T_1 \qquad (8)$$

with m_1 in terms of solar masses.

This relation was derived from the data given in ALLEN (8) for main sequence stars; it led to a determination of the mass of the main star.

In a like manner, the following relation also appeared well verified:

$$\log R_1 = -.02 + .79 \times \log m_1 \qquad (9)$$

where R_1 is in terms of solar radius and obtained from ALLEN (8). It was then possible to calculate the mass of the main star and from this to derive the elements of the companion:

$$m_2 = q \times m_1 \qquad R_2 = k \times R_1 \qquad T_2 \simeq T_1$$

the latter holding true for practically all systems in contact.

The calculations eventually led to the values given in table II.

<u>MQ CEP and GS LAC:</u> For both these stars the absolute elements were determined by means of an iterative calculation using as fundamental equations the mass-radius relation, the 3rd law of Kepler, the mass-temperature relation and the relation:

$$J_g/J_s = (\exp(-w_s) - 1)/ (\exp(-w_g) - 1) \qquad (10)$$

$$\text{with } w_{s,g} = 1.439/(\lambda_q \times T_{s,g}) \qquad (11)$$

and supposing that the primary star belonged to the main sequence. The m-R and m-T relations were obtained by interpolating the data given in ALLEN (8) for the zero-age main sequence stars. The calculations yielded the elements reported in table II.

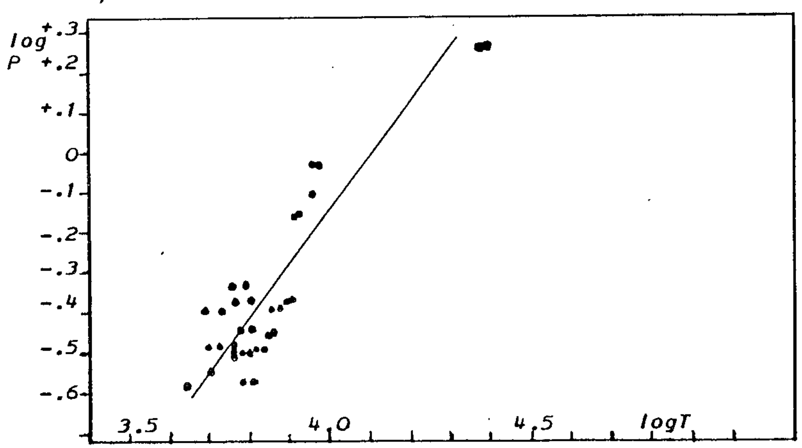

FIG. 5

TAB I   PHOTOMETRIC ORBITAL ELEMENTS

	KR LAC	MQ CEP	GS LAC	LW LAC
hypothesis at min I	tr.	occ.	tr.	occ.
α_0	1.00 ±.05	.85 ±.05	1.00 ±.03	1.00 ±.03
K	.75 ±.01	.67 ±.01	.95 ±.02	.57 ±.02
r_g	.423±.001	.381±.002	.349±.004	.201±.007
r_s	.316±.006	.257±.006	.331±.012	.114±.008
$\|i\|$	86°1 ± 1°6	78°2 ± 0°9	90°00	90°00
x_g	.51	.44	.24	.60 (ass.)
x_s	.57	.53	.27	.60 (ass.)
L_g	.81 ±.02	.312±.009	.86 ±.02	.143±.003
L_s	.19 ±.02	.688±.009	.14 ±.02	.857±.003
J_g/J_s	2.36 ±.06	.21 ±.02	5.29 ±.09	.05 ±.05

TAB II   ABSOLUTE ELEMENTS

	KR LAC	MQ CEP	GS LAC	LW LAC
T_g	10900 °K	11900 °K	19900 °K	...
T_s	10900 °K	22900 °K	10500 °K	...
m_g	3.4 M⊙	4.5 M⊙	11.7 M⊙	...
m_s	2.1 M⊙	4.6 M⊙	10.6 M⊙	...
R_g	2.4 R⊙	3.2 R⊙	5.8 R⊙	...
R_s	1.9 R⊙	2.1 R⊙	5.5 R⊙	...
a	5.67R⊙	8.4 R⊙	16.7 R⊙	...
Sp	B9+B9	B3+B8	B3+B8	...

6. CONCLUSIONS:

In the present work, orbital elements have been derived which give some idea of the nature of the stars under discussion. It is intently hoped that this work be followed by further observations which could help improving our knowledge of these interesting binary stars.

Observations should tend to a determination of the instants of minimum and the construction of a new light curve. It must kept in mind that the systems KR LAC and MQ CEP can easily show in the course of time variations either of the light curve or the period, owing to their nature. Moreover, the light curves analyzed in this work were obtained over 20 years ago! As regards LW LAC, more observations of the light curve at the primary minimum would be of interest, so as to get a better definition.

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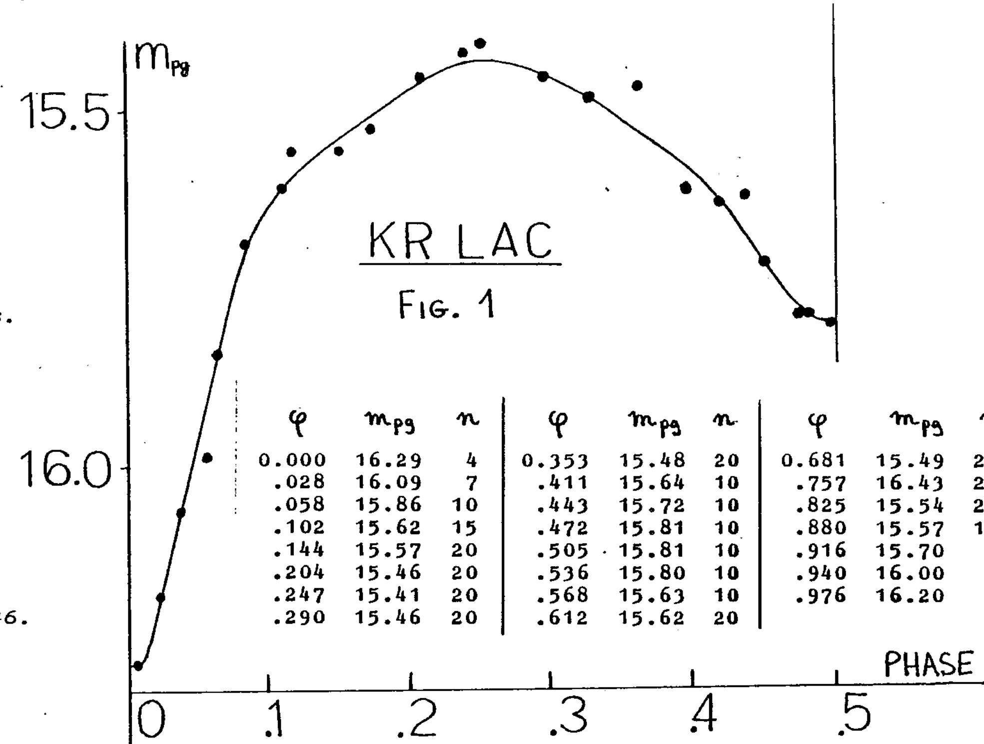
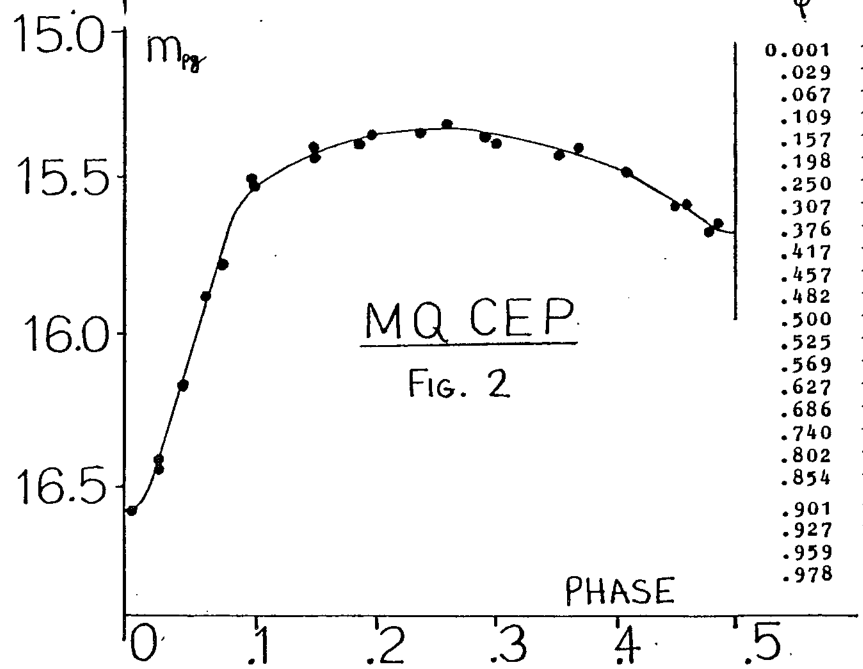


KR LAC

FIG. 1

φ	m_{pg}	n	φ	m_{pg}	n	φ	m_{pg}	n
0.000	16.29	4	0.353	15.48	20	0.681	15.49	20
.028	16.09	7	.411	15.64	10	.757	16.43	20
.058	15.86	10	.443	15.72	10	.825	15.54	20
.102	15.62	15	.472	15.81	10	.880	15.57	10
.144	15.57	20	.505	15.81	10	.916	15.70	8
.204	15.46	20	.536	15.80	10	.940	16.00	7
.247	15.41	20	.568	15.63	10	.976	16.20	7
.290	15.46	20	.612	15.62	20			



MQ CEP

FIG. 2

φ	m_{pg}	n
0.001	16.60	7
.029	16.46	7
.067	15.90	10
.109	15.54	15
.157	15.45	20
.198	15.43	20
.250	15.39	20
.307	15.43	20
.376	15.46	20
.417	15.52	20
.457	15.63	8
.482	15.72	8
.500	15.72	7
.525	15.64	10
.569	15.54	15
.627	15.49	20
.686	15.42	20
.740	15.37	20
.802	15.38	20
.854	15.44	20
.901	15.56	15
.927	15.82	12
.959	16.20	10
.978	16.47	10

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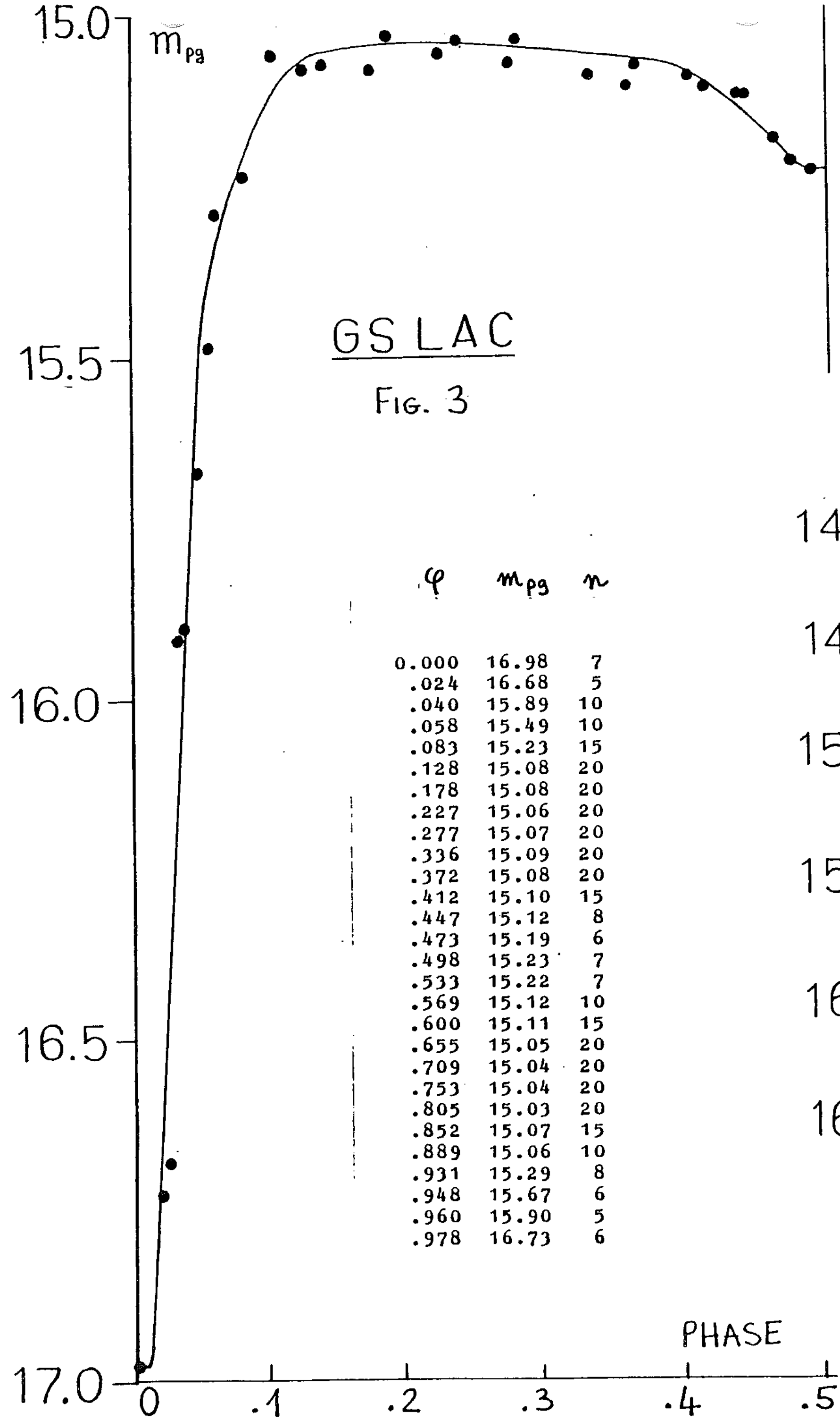
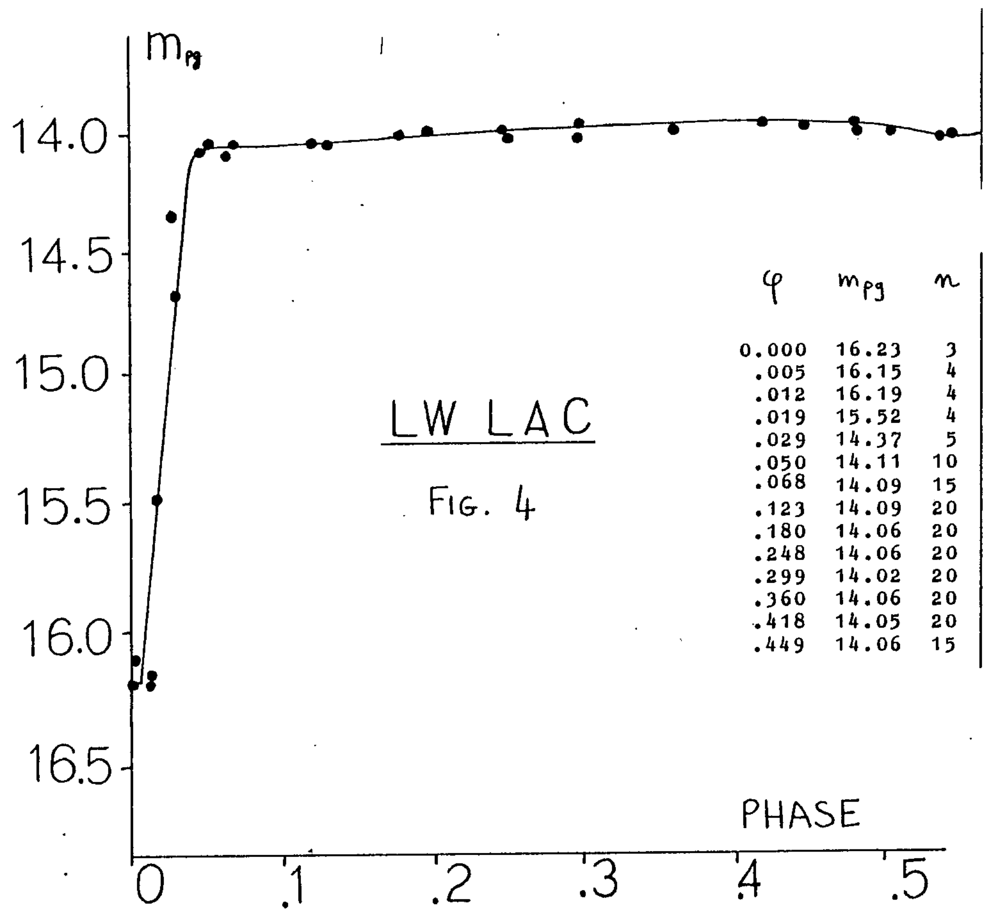


GS LAC

FIG. 3

φ	m_{pg}	n
0.000	16.98	7
.024	16.68	5
.040	15.89	10
.058	15.49	10
.083	15.23	15
.128	15.08	20
.178	15.08	20
.227	15.06	20
.277	15.07	20
.336	15.09	20
.372	15.08	20
.412	15.10	15
.447	15.12	8
.473	15.19	6
.498	15.23	7
.533	15.22	7
.569	15.12	10
.600	15.11	15
.655	15.05	20
.709	15.04	20
.753	15.04	20
.805	15.03	20
.852	15.07	15
.889	15.06	10
.931	15.29	8
.948	15.67	6
.960	15.90	5
.978	16.73	6



LW LAC

FIG. 4

φ	m_{pg}	n	φ	m_{pg}	n
0.000	16.23	3	0.483	14.07	15
.005	16.15	4	.506	14.08	15
.012	16.19	4	.543	14.08	15
.019	15.52	4	.580	14.09	20
.029	14.37	5	.636	14.05	20
.050	14.11	10	.704	14.07	20
.068	14.09	15	.748	14.06	20
.123	14.09	20	.803	14.04	20
.180	14.06	20	.867	14.08	20
.248	14.06	20	.936	14.11	10
.299	14.02	20	.949	14.09	7
.360	14.06	20	.973	14.68	3
.418	14.05	20	.987	16.23	3
.449	14.06	15			